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Livsmedelsverket • Finnish Food Authority

Uncertainty and variability in Bayesian inference for dietary risk: Listeria in RTE fish

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Listeria in Ready To Eat (RTE) Fish: Cold Smoked Salmon & Salt Cured Salmon, (CSS/SCS).

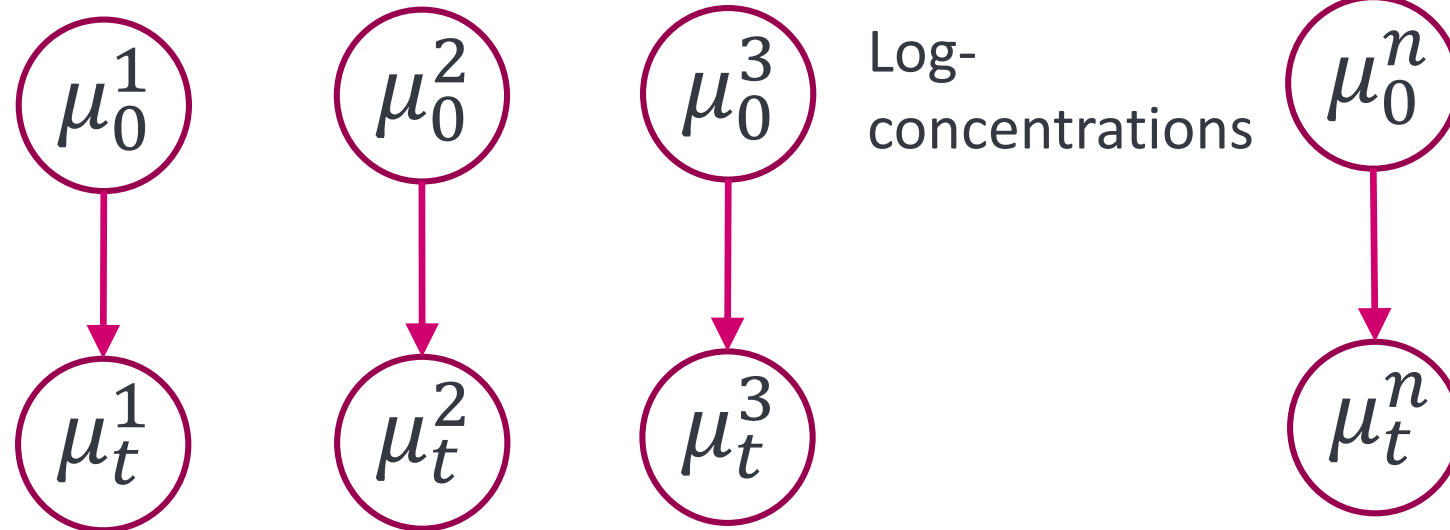
- (1) Concentration data & growth model**
 - (2) Consumption data**
 - (3) Bayesian inference**
 - (4) Dose-response model**
 - (5) Epidemiologic data: reported cases & population data, age groups.**
-



(1) Variation between products μ_0 + growth: $\mu_t = \text{growth}(\mu_0)$, t days



https://www.cookipedia.co.uk/recipes_wiki/File:Ikea_Gravadlax.jpg



q = prevalence of
positive CSS/SCS
 $\approx 22\%$

$\sim N(\mu, \sigma^2)$
43 values,
179 were <LOQ

Logistic growth model



(2) Variation between consumptions: 48h food diaries

- Log-serving sizes $\sim N(\eta, \delta^2)$.
- Consumption on a day, given consumption on previous day:
 $P(\text{yes} | \text{yes}) = p_{11}$. Likewise p_{00} .
- Consumption on a day: two-state Markov chain stationary probability p_1 .

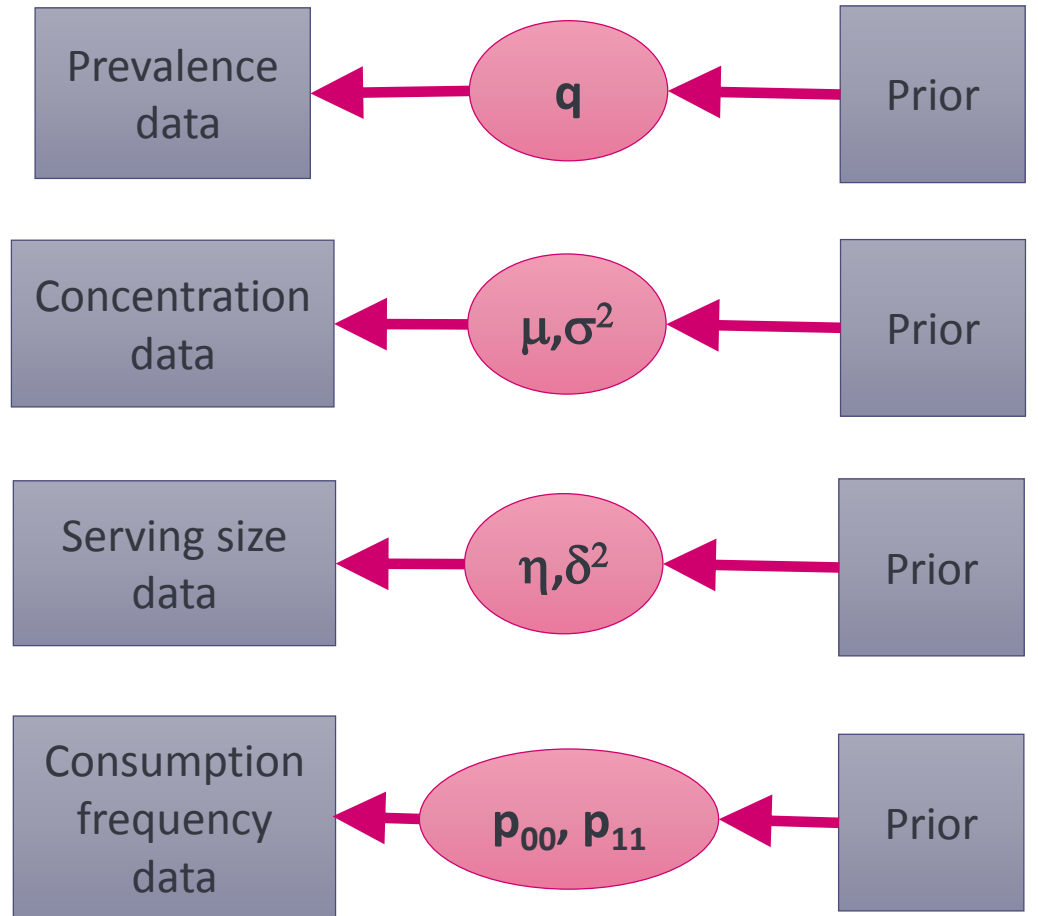
Each of the above have uncertain parameters:

- Core population parameters: $\theta = (q, \mu, \sigma^2, \eta, \delta^2, p_{00}, p_{11})$.



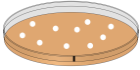

(3) Bayesian inference: $P(\theta | \text{data})$ uncertainty of population parameters

- Listeria prevalence in CSS/SCS (q)
 - Uncertain due to sample size, method accuracy.
- Concentration distribution: (μ, σ^2)
 - Uncertain due to sample size, and many values <LOQ.
- Serving size distribution: (η, δ^2)
 - Uncertain due to sample size, stratification by age.
- Consumption frequencies: transition probabilities (p_{00}, p_{11})
 - Uncertain due to rare occasions, stratification by age.





(4) Conditional dose-response probability, given consumption of contaminated CSS/SCS & parameter r

- $P(\text{illness} \mid r, E(d)) = 1 - \exp(-rE(d))$ $\{d \sim \text{Poisson}(E(d))\}$
- $E(d) = \exp(\mu_t^* + s^*) = \exp(g_t(\mu_0^*) + s^*)$
- μ_t^* = predicted log-concentration on day t , $\mu_t^* = g_t(\mu_0^*)$,  predicted initial value μ_0^* .
- s^* = predicted log-consumption amount, if consuming. 
- μ_0^* , s^* predicted from the distributions: $f(\mu_0^* \mid \mu, \sigma^2)$, $f(s^* \mid \eta, \delta^2)$, conditional on the uncertain $\mu, \sigma^2, \eta, \delta^2$



(4) Conditional probability to acquire illness, allowing repeated consumptions

- Probability to start consuming, purchase of CSS/SCS.
- Probability to continue next day, same product.
- Chance of acquiring illness conditionally on 'still at risk' & exposure on a day.
- **Total probability of illness**, over several days, allowing repeated use:



$P(\text{illness} | \mathbf{r}, \theta) =$

$$(1 - p_1)p_{01}q \left[P_1(\text{ill} | \mathbf{r}, \mu, \sigma, \eta, \delta) + \sum_{t=2}^7 \prod_{i=1}^{t-1} (1 - P_i(\text{ill} | \mathbf{r}, \mu, \sigma, \eta, \delta)) p_{11}^{t-1} P_t(\text{ill} | \mathbf{r}, \mu, \sigma, \eta, \delta) \right]$$

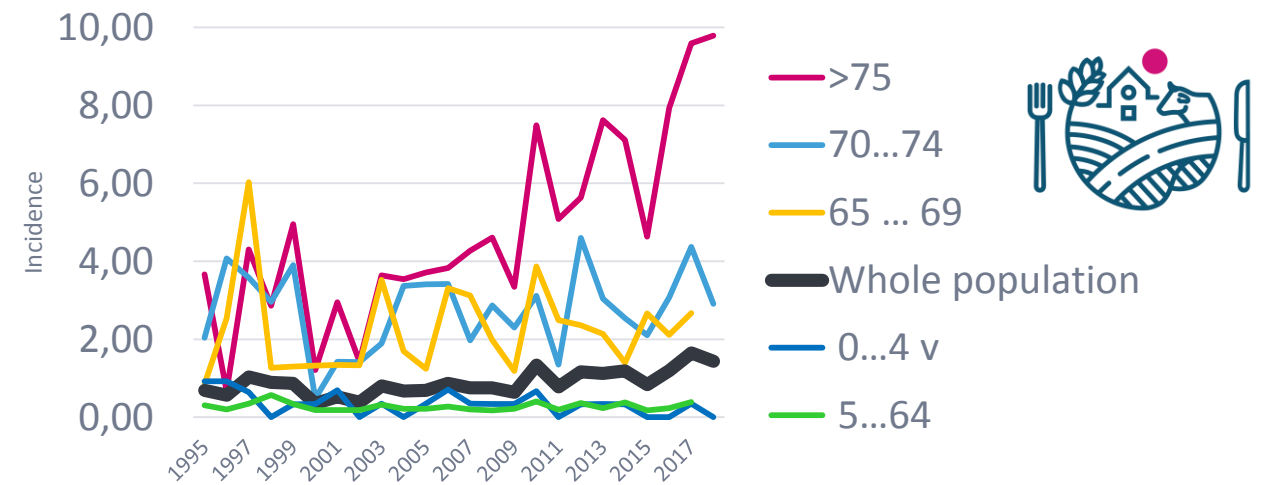
- (Age group specific).



(4) Population illness probability (risk), individual variability integrated

- Accounting for individual variability in μ_t^*, s^* requires integration:
- $P_t(\text{illness} | \mathbf{r}, \mu, \sigma, \eta, \delta) = E(P_t(\text{illness} | \mathbf{r}, g_t(\mu_0^*), s^*)) = \iint_{-\infty, -\infty}^{\infty, \infty} (1 - \exp(-\mathbf{r} \exp(g_t(\mu_0^*) + s^*))) f(\mu_0^* | \mu, \sigma) f(s^* | \eta, \delta) d\mu_0^* ds^*$
- This may have no analytic solution, but a Monte Carlo approximation:
- $\hat{P}_t(\text{illness} | \mathbf{r}, \mu, \sigma, \eta, \delta) \approx \sum_{k=1}^K (1 - \exp(-\mathbf{r} \exp(g_t(\mu_0^{*k}) + s^{*k}))) / K$
where μ_0^{*k}, s^{*k} are sampled from $f(\mu_0^* | \mu, \sigma)$ and $f(s^* | \eta, \delta)$.

(5) Epidemiologic data



- Unknown dose-response parameter r for specific age groups.
 - Uncertain due to lack of detailed epidemiological data, stratification by age.
- Using the reported cases as epidemiological data in the model.
 - Proportion of cases due to CSS/SCS? $0 \leq \text{cases}_{\text{age}} \leq \text{total}_{\text{age}}$.
 - Could use source attribution modelling, expert opinion, scenario assumption.
 - Reported cases around **12** in both 65-74 and 25-64 year olds, annually.
 - Population sizes about **470,000** vs **2,900,000**.
 - So we know *something* about incidence. → Use this in the model.
- Actually, published estimates of r rely on some back-calculations, or 'adjusting' predictions with reported incidence.

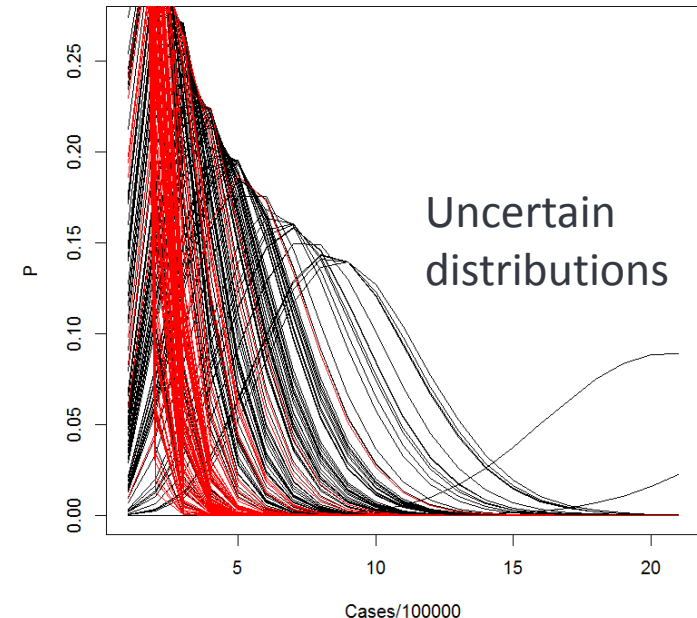


(5) Full model Bayesian inference

- Full posterior density from the model, all parameters \boldsymbol{r}, θ , formally:

$$P(\boldsymbol{r}, \theta | \text{concentration \& consumption data, cases, popula}) \propto \hat{P}(\text{cases} | \boldsymbol{r}, \theta, \text{popula}) P(\text{concentration \& consumption data} | \theta) P(\boldsymbol{r}, \theta)$$

- Where $\hat{P}(\text{cases} | \boldsymbol{r}, \theta, \text{popula}) = \text{Poisson}()$ is based on Monte Carlo approximation of the population risk within each MCMC iteration.
 - Intractable likelihood function.
 - Also denoted "2D" Monte Carlo, or MC within MCMC.
 - Increases computational burden.





Unquantified uncertainty

- Growth model with fixed parameters?
 - No home storage data.
 - Assumed temperatures as scenarios.
- Unevenly distributed, clustered microbes, mixing?
 - No data.
- Variable susceptibility among consumers?
 - Can only relate exposure and incidence data by main age-groups.
- Unknown size of purchased packages?
 - Total number of servings?
- Majority of consumptions were at home, but not all.
- Not all cases due to CSS/SCS, although major risk. Source attribution, under reporting.



Quantify as much uncertainty & variability as you can!

(while keeping it simple, feasible, evidence based...)

- This was easy →



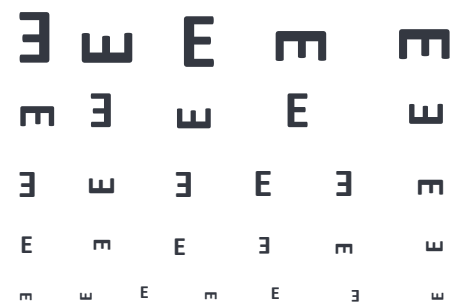
- This was possible →



- It gets harder here →



-Uhhhh →



← but some of these *could still be* important.



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Thank you!

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